

Algebra 2 Notes

Key.

2.1: Intro to Quadratic Graphs

Objectives: Define, identify, and graph simple quadratic functions in standard form. Find maximum and minimum values.

A Quadratic is a function that can be written in the form $y = ax^2 + bx + c$ where $a, b,$ and c are real numbers and $a \neq 0$. Ex: $y = 5x^2$ $y = -2x^2 + 7$ $y = x^2 - x - 3$

Quadratic Function in vertex form: $f(x) = a(x - h)^2 + k$ \rightarrow translated form

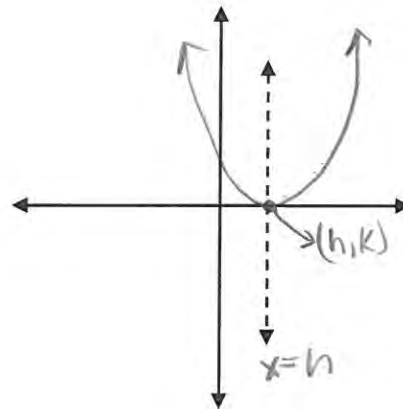
$a =$ stretch, $a < 0$ reflects
 $h =$ horizontal translation
 $k =$ vertical translation

\triangleright Graph of a quadratic function is a parabola with:

axis of symmetry $x = h$ dotted line
coordinates of the vertex (h, k)
y-int: set $x = 0$, solve
x intercept: set $y = 0$, solve

Domain: $(-\infty, \infty)$

Range: $[k, \infty)$ or $(-\infty, k]$ if reflected



Maximum and Minimum:

If $a > 0$, the parabola opens up and the y-coordinate of the vertex is the minimum of f .

If $a < 0$, the parabola opens down and the y-coordinate of the vertex is the maximum of f .

Example 1: Graph. $f(x) = -\frac{1}{2}(x+2)^2$

Identify a, h, k, $a = -\frac{1}{2}$ $h = -2$ $k = 0$

Find the axis of symmetry $x = -2$

Find the vertex $(-2, 0)$

Ask yourself, will the parabola open up or down? ↓

Is the vertex a max or min? max

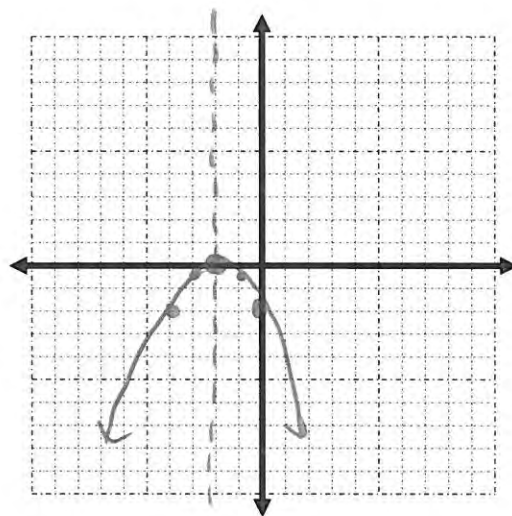
Find the y-intercept $(0, -2)$

Find the x intercepts: $(-2, 0)$

→ $f(0) = -\frac{1}{2}(0+2)^2 = -\frac{1}{2}(4) = -2$

$0 = -\frac{1}{2}(x+2)^2$

$0 = (x+2)^2$ $x+2=0$ $x=-2$



Make a table with numbers above and below the vertex if needed. Use symmetry.

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

x	y
-1	-1/2

You try: $f(x) = 2(x-1)^2 - 3$

$a=2$ $h=1$ $k=-3$

$x=1$

$f(0) = 2(0-1)^2 - 3$

$= 2 - 3 = -1$

$0 = 2(x-1)^2 - 3$

$3 = 2(x-1)^2$

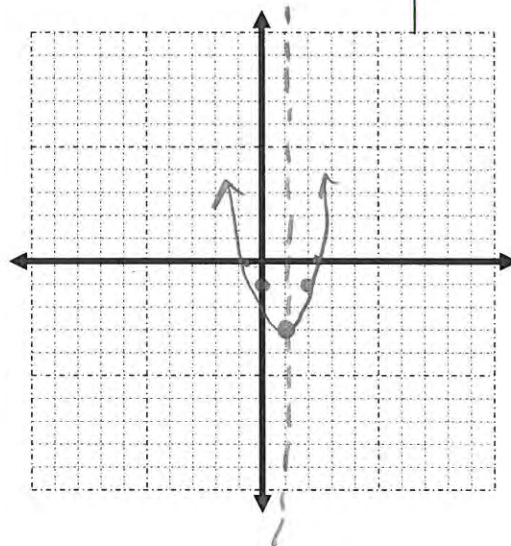
$\frac{3}{2} = (x-1)^2$

$\pm\sqrt{3/2} = x-1$

$x \approx 2.22$

$x \approx -0.22$

$x-1 = \pm 1.22$
 $x = \pm 1.22 + 1$



Example 2: Determine the key features.

$f(x) = 2(x-3)^2 + 4$

Opens: ↑

Axis of symmetry: $x=3$

Vertex: $(3, 4)$

Domain: $(-\infty, \infty)$

Range: $[4, \infty)$

x Intercepts: $0 = 2(x-3)^2 + 4$
 $-4 = 2(x-3)^2$
 $-2 = (x-3)^2$
 $x \sqrt{-2} = x-3$ no int.

y intercept: $f(0) = 2(0-3)^2 + 4$
 $2(9) + 4$
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Is the vertex a max or min? min

Example 3: Write an equation of a parabola given a vertex and point.

$$y = a(x-h)^2 + k$$

$$y = a(x+2)^2 + 3$$

Find a
 $(0, -1)$
 x y

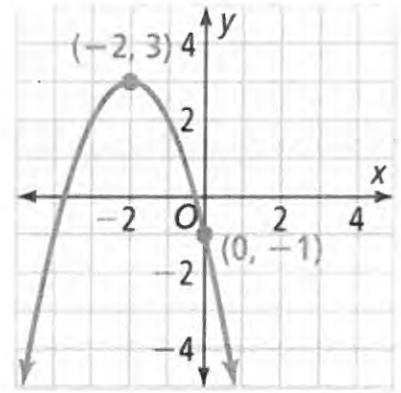
$$-1 = a(0+2)^2 + 3$$

$$-1 = 4a + 3$$

$$-4 = 4a$$

$$-1 = a$$

rewrite $y = -1(x+2)^2 + 3$



You try. What is the equation of a parabola with a vertex of $(1, -4)$ and which passes through $(-2, -1)$?

$$y = a(x-1)^2 - 4$$

$$-1 = a(-2-1)^2 - 4$$

$$-1 = a(-3)^2 - 4$$

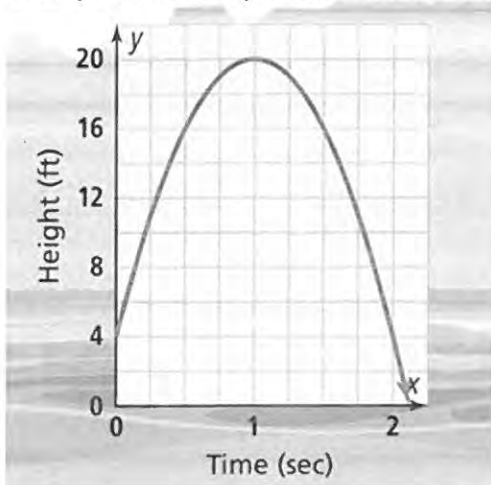
$$3 = 9a$$

$$\frac{1}{3} = a$$

$$y = \frac{1}{3}(x-1)^2 - 4$$

Example 4: Modeling height and writing in standard form.

The height of a thrown ball is a quadratic function of the time it has been in the air. The graph of the quadratic function is the parabolic path of the ball. The vertex of the graph is $(1, 20)$ and the path of the ball includes the point $(0, 4)$. What is an expression that defines this function? Write the quadratic equation in vertex form and in the form $y = ax^2 + bx + c$



$$y = a(x-1)^2 + 20$$

$$4 = a(0-1)^2 + 20$$

$$-16 = a$$

$$y = -16(x-1)^2 + 20$$



FOIL

$$y = -16(x-1)(x-1) + 20$$

$$= -16(x^2 - x - x + 1) + 20$$

$$= -16(x^2 - 2x + 1) + 20$$

$$= -16x^2 + 32x - 16 + 20$$

$$= -16x^2 + 32x + 4$$

To change to standard form:

You try. The diagram shows the path of a model rocket launched from the ground. It reaches a maximum altitude of 384 ft when it is above a location 16 ft from the launch site. What quadratic function models the height of the rocket?

$$y = a(x - 16)^2 + 384$$

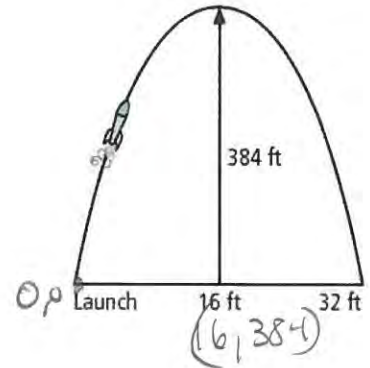
$$0 = a(0 - 16)^2 + 384$$

$$0 = 256a + 384$$

$$\frac{-384}{256} = a$$

$$-3/2 = a$$

$$y = -3/2(x - 16)^2 + 384$$



Example 5. Write an equation for a transformed function.

The function g is a translation of the parent function f 1 unit left and 3 units up and no vertical stretch. What is the equation of g ? Write the quadratic equation in vertex form and in the form

$$y = ax^2 + bx + c$$

$$g(x) = (x + 1)^2 + 3$$

$$g(x) = (x + 1)(x + 1) + 3$$

$$= x^2 + 2x + 1 + 3$$

$$g(x) = \boxed{x^2 + 2x + 4}$$